Peasant Multiplication **Danzig p 26, Reid pp 32-34**

There is a story of a German merchant of the fifteenth century, which I have not succeeded in authenticating, but it is so characteristic of the situation then existing that I cannot resist the temptation of telling it. It appears that the merchant had a son whom he desired to give an advanced commercial education. He appealed to a prominent professor of a university for advice as to where he should send his son. The reply was that if the mathematical curriculum of the young man was to be confined to adding and subtracting, he perhaps could obtain the instruction in a German university; but the art of multiplying and dividing, he continued, had been greatly developed in Italy, which in his opinion was the only country where such advanced instruction could be obtained.

As a matter of fact, multiplication and division as practiced in those days had little in common with the modern operations bearing the same names. Multiplication, for instance, was a succession of *duplations*, which was the name given to the doubling of a number. In the same way division was reduced to *mediation*, i.e., "halving" a number. A clearer insight into the status of reckoning in the Middle Ages can be obtained from an example. Using modern notations:

Today	Thirteenth century
46	$46 \times 2 = 92$
13	$46 \times 4 = 92 \times 2 = 184$
138	$46 \times 8 = 184 \times 2 = 368$
46	$368 + 184 + 46 = 598$
598	

But there's rather more to it than this, and involves the implicit use of binary arithmetic, which was devised by Gottfried Leibniz in the seventeenth century, though the methodology goes back to the Ancient Egyptians.

https://www.cuemath.com/numbers/decimal-to-binary/

https://mathcurious.com/2019/12/17/ancient-methods-of-multiplication-the-egyptian-form-of-multiplication/

https://math.libretexts.org/Courses/Mount_Royal_University/MATH_2150%3A_Higher_Arithmetic/0%3A_Preliminaries/0.4%3A_Egyptian_Multiplication_and_Division_(optional)

As you probably know, the **decimal number** system is a positional number system with base 10, using 10 symbols : 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Decimal numbers can be written as (eg) 45_{10}, 118_{10}, and so on. It is the most commonly known number system in which the numbers can be identified easily even if the base is not written - if the base of a number is not specified, it is considered to be a decimal number.

In a decimal number, the digit to the extreme left is called the Most Significant Digit (MSD), and the digit to the extreme right is known as the Least Significant Digit (LSD).

But as you may not yet know, the **binary number** system is a positional number system with base 2, using only two digits, 0 and 1. The smallest unit of data in a computer is called a bit, which is the abbreviated form of 'binary digit'. Binary numbers can be written as (eg) 110_2, 10_2 for emphasis

In a binary number, the bit to the extreme left is called the Most Significant Bit (MSB), and the bit to the extreme right end is known as the Least Significant Bit (LSB).

There is a well-established procedure for interconversion of any number from one base to any other another, as exemplified by the following example from decimal to binary.

1. The trial number (29) is divided repeatedly by 2 until the quotient 0 is reached, the remainders being recorded in order at each step.

29 ÷ 2 = 14 remainder 1

14 ÷ 2 = 7 remainder 0

7 ÷ 2 = 3 remainder 1

3 ÷ 2 = 1 remainder 1

1 ÷ 2 = 0 remainder 1

2. The remainders are written in their reverse order.

$29_{10} = 11101_2$ (16 + 8 +4 + 0 + 1)

3. Note that a remainder of zero invariably correlates with a zero in the binary result.

To quote more or less directly from Constance Reid :

“[By the seventeenth century] the principle of representation by powers of two was [unknowingly] used by people who would never have recognised a number expressed in the binary system. These people [the rural peasantry], who knew so little of arithmetic that they could not even multiply except by two [‘duplation’], or divide except by two [‘mediation’, flooring any fractions that arise] had worked out [or inherited] their own very neat system of multiplying any number in this way.

The system works like this.

1. To multiply 29 by 31, divide 29 by 2 and the answer again by 2 and so on until you have only a remainder of 1.

/continued

2. Then double 31 the same number of times that you have halved 29, keeping halvings and doublings, crossing out whatever doubling occurs opposite an even halving.

3. Total the remaining doublings to obtain your answer.

29	31
14	~~62~~
7	124
3	248
1	496
	899

If you multiply 29 and 31 in the customary way, you will obtain the same answer."

She gives fairly short shrift to the emergence of binary notation, where I would give it top billing

$29_{10} = 11101_2 = 1 + 4 + 8 + 16$

So $29_{10} \times 31 = (1 \times 31) + (4 \times 31) + (8 \times 31) + (16 \times 31)$

$= 31 + 62 + 124 + 248 + 496 - 62$

$= 899$

I suspect that any zeros were retained, to sustain the flow of doubling, and then subtracted at the end.

The example provided by Danzig can now be set out in standard fashion

Mediation	Duplation
13 x	**46**
13	46
6	92
3	184
1	368
	= 598

= 598

And indeed more complicated examples can be dealt with similarly

Mediation	Duplation
37 x	**146**
37	146
18	292
9	584
4	1168
2	2336
1	4672
	= 5402

= 5402