Water and Wine

At least half of G K Chesterton's œuvre is incomprehensible to the modern reader, but much of it is soul-shakingly inspired, and many of his aphoristic insights have passed into common parlance – he could invariably perceive the eternal in the commonplace, such as the closing line '[Man may] get to Paradise by way of Kensal Green' to his poem *The Rolling English Road*, in which he famously celebrated inebriation as a means of communion with the divine.

And in his poem *Wine and Water*, the refrain tells us 'But I don't care where the water goes if it doesn't get into the wine'. And that, very conveniently, gives point to the brain-teaser that follows, in which, in tribute to the immortal G K, I've abandoned the water and used red wine and white wine instead (a head-splitting combination, please don't try to neck the contents of A and B afterwards).

Suppose you have two measuring jugs A and B, and you pour 100 cc of red wine into jug A, and 100 cc of white wine into jug B. Suppose then that with a pipette you transfer 10 cc of red wine from jug A to jug B, and allow it to mingle thoroughly. And suppose finally that with your pipette you transfer 10 cc of the mixture from jug B to jug A.

The question is whether there is now more red wine in jug B that white wine in jug A, or vice versa, if either ? The figures quoted are entirely arbitrary and don't affect the answer in any way.

Step	A	B	Σ
1	$R_A = 100$ $W_A = 0$	$R_B = 0$ $W_B = 100$	$R_A + R_B = 100$ $W_A + W_B = 100$
2	$R_A = 90$ $W_A = 0$	$R_B = 10$ $W_B = 100$	$R_A + R_B = 100$ $W_A + W_B = 100$
3	$R_A = 90 + r.10$ $W_A = w.10$	$R_B = 10 – r.10$ $W_B = 100 – w.10$	$R_A + R_B = 100$ $W_A + W_B = 100$
Σ	$R_A + W_A = 90 + 10.(r + w)$ $= 100$	$R_B + W_B = 110 –10.(r + w)$ $= 100$	

In Step 3, the relative proportion of red wine in the 10 cc transferred to back to Jug A is symbolised by r, and the complementary proportion of white wine transferred is symbolised by w.

Clearly, $r = 10/110$ and $w = 100/110$, so that $r + w = 110/110 = 1$

The red wine left in jug B is symbolised by $R_B = 10 – r.10 = 10.(1 – r)$

And the white wine introduced to jug A is likewise $W_A = w.10 = 10.(1 – r)$

So in fact, counter-intuitively, there will be exactly the same amount of red wine in the white wine as the amount of white wine in the red wine – a state of affairs that would surely have appealed to GKC.

The initial quantity of wine in each of the two jugs is of course immaterial, as is the quantity transferred in each of Step 2 and Step 3, and could be symbolised by V and v respectively. Which makes me suspect that the details are almost incidental, and that it's possible to see the solution intuitively.

Well, as close to intuitive as I can get, are two of the conservation (Σ) statements

$$R_A + R_B = 100$$
$$R_A + W_A = 100$$

so that $R_B = W_A$!!!