

THE MULTIPLIED HORSE-SHOE NAILS
from
Sandford & Merton, by Thomas Day (1748-1789)

As reproduced in *The Patchwork Book*,
compiled and introduced by Marghanita Laski,
Pilot Press, London 1946, pp295-7

There was a gentleman who was extremely fond of beautiful horses, and did not grudge to give the highest price for them. One day a horse-courser came to him, and showed him one so handsome that he thought it superior to all he had ever seen before. He mounted him, and found his paces equally excellent; for, though he was full of spirit, he was gentle and tractable as could be wished. So many perfections delighted the gentleman, and he eagerly demanded the price. The horse-courser answered that he would abate nothing of two hundred guineas; the gentleman, although he admired the horse, would not consent to give it; and they were just on the point of parting. As the man was turning his back, the gentleman called out to him, and said, "Is there no possible way of our agreeing ? for I would give you anything in reason for such a beautiful creature."

"Why," replied the dealer, who was a shrewd fellow, and perfectly understood calculation, "if you do not like to give me two hundred guineas, will you give me a farthing for the first nail the horse has in his shoe, two farthings for the second, four for the third, and so go doubling throughout the whole twenty-four ? for there are no more than twenty-four nails in all his shoes." The gentleman gladly accepted the condition, and ordered the horse to be led away to his stables.

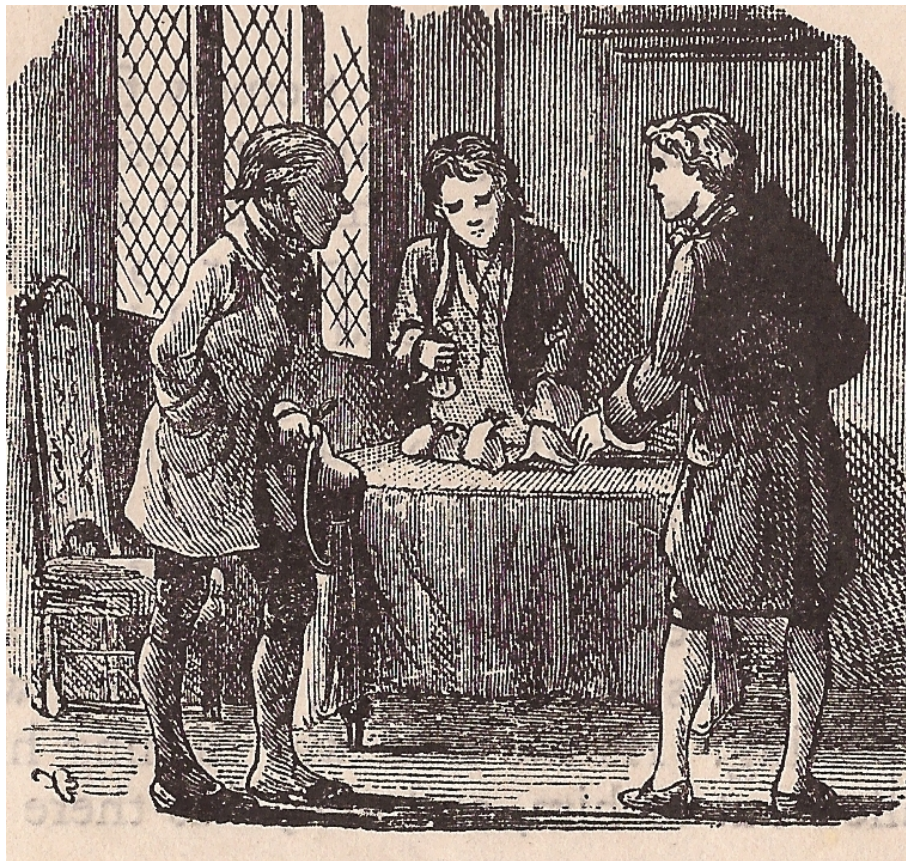
However the horse-courser added, "I do not mean, sir, to tie you down to this last proposal, which, upon consideration, you may like as little as the first; all that I require is that, if you are dissatisfied with your bargain, you will promise to pay me down the two hundred guineas which I first asked."

This the gentleman willingly agreed to, and then called the steward to calculate the sum, for he was too much of a gentleman to be able to do it himself. The steward sat down with his pen and ink, and after some time gravely wished his master joy, and asked him, "in what part of England the estate was situated that he was going to purchase ? "

"Are you mad ?" replied the gentleman: "it is not an estate, but a horse, that I have just bargained for; and here is the owner of him to whom I am going to pay the money."

"If there be any madness, sir," replied the steward "it certainly is not on my side : the sum you have ordered me to calculate, comes just to seventeen thousand four hundred and seventy-six pounds, besides some shillings and pence : and surely no man in his senses would give this price for a horse."

The gentleman was more surprised than he had ever been before, to hear the assertion of his steward; but, when, upon examination, he found it to be no more than the truth, he was very glad to compound for his foolish agreement, by giving the horse-courser two hundred guineas, and dismissing him.



There have been many variations on this theme from time immemorial, and countless cultures, those from the Orient generally involving grains of rice, or wheat, on an 8x8 chess-board – thereby involving a further forty doublings !

The mathematics in this case relate to the 24th partial sum of a geometrical progression with first term 1 and common ratio 2, the answer being expressed in farthings (I can remember farthings, beautiful little bronze coins with a plump wren (of the avian rather than naval persuasion) on the reverse, which were withdrawn from circulation in 1956). Their face value was a quarter (ie fourthing, as per the old subdivisions of Yorkshire into thirds ie thriddings) of a pre-decimal penny, of which latter there were 240 to the pound.

So there would be 960 farthings to the pound. In my own pre-decimal generation, as in the narrative above, conversion of 'pounds shillings and pence' to decimal format, or vice-versa, always involved farthings as an intermediate. And there were no electronic calculators then ! Though in the commercial and financial sectors there were printed 'ready-reckoners' to help things along.

Perhaps it was purely coincidental that our national decline into embarrassing ineptitude and the abandoning of our traditional multiple-base currency (and weights and measures) seemed to go hand in hand. I'm rather more than half serious that the arcane complexities of these apparent anachronisms had until then kept our collective wits several mental jumps ahead of our foreign competitors.

Be all that as it may, how did the steward put a figure to the horse-courser's suggestion ? The exact amount in farthings was of course

$$2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^{23} = \frac{2^{23} - 1}{2 - 1} \sim 2^{23}$$

So that the amount in pounds was

$$\pounds \frac{2^{23}}{960} = \pounds \frac{2^{23}}{3 \times 5 \times 2^6} = \pounds \frac{2^{17}}{15} = \pounds \frac{262144}{15} = \pounds 17,476.26$$

just as the steward pronounced. As for the shillings and pence,

$$\pounds 0.26 = 0.26 \times 240d = 62.4d = 5s 2.4d \sim 5/2\frac{1}{2}d$$

(ie five and tuppence ha'penny, which was probably an agricultural labourer's weekly wage at that time, give or take a few pence)

One minor mystery remains – what were the ‘guineas’ in which the horse-courser's opening price was expressed ? Twenty-one shillings is the quick and easy answer, and was (and to some extent remains) the unit in which the professions invoiced their clients, and in which the gentry and aristocracy transacted their purchases. But it was a lot more complicated than that, as Wikipedia will confirm.