Clock Calculations
[This was in reply to a query from Merwyn Colaço]

(Formula A) For linear motion, distance travelled = linear speed * time taken where linear speed is measured in miles per hour (say)

(Formula B) For circular motion, distance travelled = angular speed * time taken where angular speed is measured in radians per hour (say) remembering that 2 * π radians = 360 degrees

The minute hand turns through 360 degrees in 1 hour
so its angular speed ω_1 = 2 * π (radians/hour)

The hour hand turns through 360 / 12 = 30 degrees in 1 hour
so its angular speed ω_1 = 2 * π / 12 = π / 6 (radians/hour)

so the angular speed of the minute hand <u>seen from the hour hand's point of view</u> is 2 * π - π / 6 = 11 * π / 6 (radians/hour)

Thus the angle that develops between the hour hand and the 'run-away' minute hand over a period of h hours is ϑ = (11 * π / 6) * h , as per Formula B above.

Now, remember that cos (0) = 1 and cos (π/2) = 0 and cos (π) = -1 and cos (3π/2) = 0

0 corresponds to coincident hands, π/2 and 3π/2 corresponds to perpendicular hands, and π to opposing hands.

So for coincident hands, cos (11π h/ 6) = cos (0) = 1
& for perpendicular hands, cos (11π h/ 6) = cos (π/2 or 3π/2) = 0
& for opposing hands, cos (11π h/ 6) = cos (π) = -1

Let's focus on perpendicular hands, and think back to the cos(ϑ) graph from the Good Old Days: cos(ϑ) = 0 for π/2, 3π/2, 5π/2, 7π/2, 9π/2, ...

ie for multiples 1, 3, 5, 7, 9, ... of π/2
ie for multiples 2n +1 of π/2 where n = 0, 1, 2, 3, 4, ...

so 11π h/ 6 = (2n + 1) * π/2

ie h = 3*(2n + 1) / 11 are the times for which the hands are perpendicular.

By inspection, h is just over 24 for n = 44, which is therefore disallowed. So n = 0, 1, 2, 3, 4, ... , 43, <u>a total of 44 occasions</u>.