

## The Shared Birthday paradox [This was in reply to a query from Quenton Steele]

If there are more than 23 people in a room, it's more likely than not that at least two of them share a birthday.

The following argument does ignore the existence of Feb 29 birthdays, but let's go along with this simplification.

P(something) simply means the probability of 'something' being the case.

Let's say there are N people in a room: label them as

$$\{A, B, C, D, \dots, M = (N-1), N\}$$

1) A :  $P(A) = 1$

2) B :  $P(A \ \& \ B \text{ differ}) = 364/365$

3) C :  $P(A \ \& \ B \ \& \ C \text{ differ}) = 364/365 \times 363/365;$   
[nb 2 factors]

4) D :  $P(A \ \& \ B \ \& \ C \ \& \ D \text{ differ}) = 364/365 \times 363/365 \times 362/365$   
[nb 3 factors]

...  
...

N) N :  $P(A \ \& \ B \ \& \ C \ \& \ D \ \& \dots \ \& \ N \text{ differ}) =$   
 $364/365 \times 363/365 \times 362/365 \times \dots \times (365-M)/365$   
[nb (N-1) factors]

So the probability that they don't all differ, i.e. at least two are same, is

$$1 - (364/365 \times 363/365 \times 362/365 \times \dots \times (365-M)/365)$$

If you work through the product (alternate divisions and multiplications) for N=22 [NB make a note of this value !!!] and subtract from one, the answer should be just less than 0.5 (i.e. <50%), and if you do the same for N=23 [reusing the figure already noted] the answer should be just greater than 0.5 (i.e. >50%),